Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

METHODS OF MEASUREMENT AND REDUCTION OF SPECTROGRAMS FOR THE DETERMINATION OF RADIAL VELOCITIES.

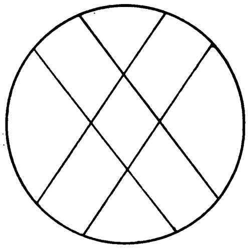
By J. H. Moore.

There is probably no field of astronomy in which the development of instruments and methods has been so rapid as in the application of the spectroscope to the determination of the radial velocities of stars according to the Doppler-Fizeau principle. In the first investigations made by Huggins in 1867, and by Vogel in 1872, the observations were visual, and it was not unusual to obtain discrepancies in the results amounting to as much as the quantities to be measured. Observations of value date from the application, by Vogel in 1887, of the photographic method to this problem, a method which had already achieved remarkable results in the hands of Professor Pickering in his qualitative studies of stellar spectra. Following this and the subsequent improvement in design of spectrographs and methods of reduction in which Professor Campbell at the Lick Observatory led the way, there has been a continual development in the instrumental and observational side giving spectrograms of greater accuracy. At the same time refinements have been introduced into the methods of measuring and reducing these spectrograms in order to obtain from them velocities as accurate as the plates will warrant. It will be the purpose of the present article to outline the various methods which have been used for the measurement and reduction of spectrograms for radial velocity determinations and to discuss the advantages and limitations of each.

A stellar spectrogram for this purpose consists of a star spectrum and a comparison spectrum produced by a known source, such as the metallic arc, spark, or vacuum-tube discharge. The light from the star and the artificial source are made to pass over equivalent optical paths in the spectrograph and the spectra of the two brought to a focus on the photographic plate in such a way that the star spectrum falls in the center and the comparison spectrum on each side of it. In most astronomical spectrographs of high dispersion only a comparatively short section of the spectrum (about three or four hundred tenth-meters) is in sharp focus. With these instruments we utilize only the part of the spectrum to which the photographic plate is extremely sensitive. Fortunately this region is as rich in lines as any other part of the spectrum, and is about the one of maximum dispersion of glass prisms consistent with permissible loss of light by absorption. In some spectrographs the prisms are set at minimum deviation for λ 4340, in others at λ 4500, or for some point intermediate between these wave-lengths. The measurable spectrum extends from about λ 4230 to λ 4490 in the first case, and from about λ 4340 to λ 4630 in the second case. For comparison spectrum a metal is chosen which is rich in lines for the region used. For example, iron is generally chosen for the former, while titanium and vanadium are employed for the latter region. The positions on the plate of the lines in the star spectrum (*i. e.* their wave-lengths) are, according to the Doppler-Fizeau principle, affected by the relative velocity of the star and the observer in the line of sight, while the wave-lengths of the comparison lines are unaffected by such velocity, their source being on the instrument. The radial velocity of the star is determined by measurement of the relative positions of the lines in the two spectra.

MEASUREMENT OF PLATES.

Most line-of-sight observers have employed for the measurement of spectrograms either the Töpfer engine, or the one designed by Professors HALE and FROST and made by GAERTNER. It is necessary first to test for the errors of the screw, and if they are comparable with the error of setting on lines of the spectrum they must be taken into account. Most observers use one vertical wire for making the settings. The accuracy of settings on strong comparison lines is increased, according to FROST, by the use of two parallel vertical wires, but such a form is seldom used. A reticle of the form shown in the figure, sometimes used in the measure of grating plates, possesses points of advantage over either of the other forms.



The method of measuring a plate varies somewhat with different observers, but, with the exception of the use of the Hartmann spectrocomparator, is briefly as follows: The plate is placed on the engine (usually with violet apparently left), properly aligned, and so adjusted that the reading on a selected comparison line shall be some arbitrary one agreeing with the standard table to be used for reduction. Settings are made continuously along the plate on good star and comparison lines as they chance to occur. The plate is then reversed and the settings repeated, the plate being so adjusted, for convenience, that the reading on the first comparison line in the second position shall equal a constant (say 100) minus the reading on the same line in the first position. The effect of errors due to personal equation is, according to the investigations of Professor LORD and Professor BELOPOLSKY and the more exhaustive ones of Dr. REESE,[1] practically eliminated by taking the mean of the measures of the plate in the two positions. It should be noticed, however, that in the reversal of the plate the spectrum is also inverted, which might so change the appearance of lines as to interfere with the elimination of personal equation in this way. Curvature of the spectrum lines may also interfere in like manner with the elimination of this source of error. The effect of accidental errors in setting on lines is reduced by employing a number of lines. Care should be taken to maintain the engine at as nearly a constant temperature as possible, and especially to keep the illumination of the plate the same during the entire measure.

METHODS OF REDUCTION.

Professor VOGEL in his early work used only the Hγ line for comparison, and measured the displacement between the Hγ of the star spectrum and that of the vacuum tube. Later, for stars of the solar type he employed the iron spectrum for comparison, and measured the displacement between the iron lines in the star and comparison spectra. This gave only about eight or ten independent measures of the displacement, so that in the case of stars rich in lines, as those of the F to M types, he was using only a very small part of the material on each plate and not making the best possible selection of lines. Methods have been devised, however, which enable us to use other lines in the star spectrum than those corresponding to the comparison spectrum. These may be grouped under the following heads: I. Methods of interpolation by dispersion formulæ; II. The velocity standard method; III. The spectro-comparator method.

---

[1] *L. O. Bulletin* No. 15, and *Astrophysical Journal*, Vol. 15, 208, 1902.

I. *Methods of Interpolation.*—Methods of interpolation were introduced by Professor CAMPBELL in 1896 (not published until 1898[1]) and by Professor HARTMANN in 1898,[2] which made it possible to use all the stellar lines on the plate, provided their wave-lengths are known. This is accomplished by the aid of a table of micrometer settings corresponding to the wave-lengths of the lines employed. The relation between wave-length and the corresponding micrometer setting is expressed by means of a dispersion formula, the constants of which are determined from the measures of lines of known wave-length in a spectrum of a source whose radial velocity is approximately zero,—for example, the Sun.

In explaining this method we shall first assume that it is possible to obtain a formula which shall accurately represent the relation existing between wave-lengths and micrometer readings for the whole region of spectrum to be used, and that such a table has been formed from our Sun plate, leaving to a later paragraph the discussion of the formulæ actually used by CAMPBELL and by HARTMANN.

λ	Sun.	rV_s
4338.084	10.258	299
4340.634	10.846	300
⋮	⋮	⋮
4351.930	13.433	304
⋮	⋮	⋮
4501.448	43.723	367
⋮	⋮	⋮
4656.644	69.215	434

We have, then, a table we shall call our standard dispersion table. In the first column under λ are given the wave-lengths of the lines of the Sun taken from ROWLAND'S Preliminary Table of Solar Wave-Lengths. The second column contains the micrometer readings corresponding to the wave-lengths of the first column. These settings are computed from the constants of the Sun plate. The third column contains the values of rV_s, the factor for each line by which the displacement is multiplied in order to obtain the corresponding velocity in kilometers per second; V_s, the velocity in kilometers per second corresponding to a displacement of one tenth-meter $= \dfrac{\text{velocity of light}}{\text{wave-length}} = \dfrac{299860}{\lambda}$. The quantity *r*, the number of tenth-meters in one revolution of the screw, is obtained from the original constants of the dispersion formula. Its value will be deduced later.

---

[1] *Astrophysical Journal,* Vol. 8, 123, 1898.

[2] *Astrophysical Journal,* Vol. 8, 218, 1898; *Astronomische Nachrichten,* Vol. 155, 81-118, 1901.

In measuring the star plate, set it so that one of the comparison lines—say 4338.084—has the tabular micrometer reading 10.258, and make the settings on the other comparison and star lines as described above. Take the difference, micrometer reading of comparison line on star plate minus micrometer reading of same comparison line in our table (under Sun). Next plot a curve with wave-lengths as abscissæ, and these differences as ordinates. Construct a table of micrometer settings (for zero velocity) for star lines measured on the plate by applying to the tabulated settings for these lines corrections read off from the curve. Suppose, for example, we wish to obtain the reading for the line 4351.930 on a star plate (*i. e.* the reading which this star line would have if unshifted by radial velocity). We read off from the curve the difference corresponding to λ 4351.930 and add it to the micrometer reading 13.433 in the table. Suppose it is + .022. Then the reading for zero velocity would be 13.455, and if the micrometer setting of this line on the star plate is 13.330, then the displacement is — .135. Where there are comparison lines corresponding to star lines some observers take the difference between the micrometer readings directly for the displacements, while others rigorously follow the process just outlined. Having now the displacements for each of the stellar lines measured it is necessary to multiply each one by the corresponding rV_s in order to obtain the velocity. The mean of the velocities obtained from the different lines, which we may call v_s, is the relative velocity of the star and observer in the line of sight. It is necessary to apply two small corrections to v_s; a correction for scale and a correction for curvature of the spectrum lines. If the dispersion of the star plate differs greatly from that of the dispersion table, the values of *r* in the table will be too large or small for the star plate. In place of computing the *r* for each line we apply a correction to v_s, obtained as follows: Let *a* be the difference between the tabular micrometer readings of the first and last comparison lines used, and b the difference in the actual settings on the plate for these same comparison lines, then the correction for scale $= \frac{a-b}{a} v_s$. It has the same sign as v_s if $a > b$ and opposite if $a < b$. The correction for curvature of the spectral lines may be computed by DITSCHEINER'S formula,[1] which has been found by ADAMS[2] to be accurate for long slits. Some observers determined this correction empirically from lines in the solar spectrum on the assumption that the curve is a parabola. The effect of curvature is eliminated in the new Mills spectrograph by using a curved slit which gives straight spectral lines.

In order to reduce this value of the radial velocity to the Sun, the corrections for the annual and diurnal motions of the Earth are computed from the formulæ given by Professor CAMPBELL in FROST-SCHEINER, pp. 338-345.[3] If v_a be the correction to the observed velocity of the star for the Earth's annual motion, then

$$v_a = - V_a \sin (\lambda - \odot + i) \cos \beta$$

where V_a = the Earth's velocity in kilometers per second in its orbit;

λ and β = the longitude and latitude of the star observed;

$\odot$ = the Sun's longitude at the time of observation;

$90° - i$ = the angle which the tangent to the Earth's orbit makes with the radius-vector drawn to the point of tangency.

The correction v_d due to the diurnal rotation of the Earth is given by

$$v_d = - V_d \sin t \cos \delta \cos \phi$$

where V_d = the velocity in kilometers per second of a point on the Earth's equator due to the diurnal rotation, and equals 0.47;

t = the hour angle of the star observed;

δ = the declination of the star observed;

ϕ = the latitude of the observer.

---

[1] For this formula and its derivation see FROST-SCHEINER, p. 15; also, see an article by Professor LORD, *Astrophysical Journal*, Vol. 5, 348, 1897.

[2] *Astrophysical Journal*, Vol. 11, 309, 1900.

[3] See Professor SCHLESINGER'S article in *Astrophysical Journal*, Vol. 10, 1-13, 1899; also an approximate method for reductions to the Sun by Dr. PALMER in *L. O. Bulletin* No. 98, 1906; and a recent graphical method proposed by HARTMANN in *Astronomische Nachrichten*, Vol. 173, 97, 1906.

The reduction is simplified by the use of tables giving V_a and i for different values of ⊙, and a table giving V_d for different hour angles and declinations. The value of the lunar correction is small and is usually neglected. It may amount at maximum to about .01 kilometer.

Dispersion Formulæ.—In order to determine the relation existing between wave-lengths and micrometer readings CAMPBELL[1] measured the positions of twenty-two selected lines on a plate of the solar spectrum made with the Mills spectrograph. Taking wave-lengths as abscissæ and micrometer readings as ordinates, he assumed the origin of abscissæ at λ 4330 (*i. e.* a point near the middle of the plate) and the origin of ordinates at micrometer readings 32.000 corresponding to λ 4330. If x is the difference in wave-length in tenth-meters between any line and λ 4330, he assumed that the micrometer reading of a line whose wave-length was λ $(4330 + x)$ was given by $R = 32.000 + a + bx + cx^2 + dx^3$. From the twenty-two lines as many equations resulted for the determination of the constants a, b, c, and d, which were solved by the method of least squares. The agreement between the computed and observed micrometer readings was found to be satisfactory not only for the above lines but for intermediate lines as well, throughout the whole region of spectrum employed by him (λ 4238 to λ 4442). In order to obtain the value of r in the expression rV_s, he obtained by differentiating the above formula $\frac{dx}{dR} = \frac{1}{b + 2cx + 3dx^2}$ Taking dR equal to unity dx becomes the number of tenth-meters in one revolution of the screw (*i. e.*, $r = dx$). The wave-lengths used were taken from ROWLAND'S table of solar wave-lengths, and the assumption was made that the wave-lengths of the iron comparison lines used were the same as those of the corresponding iron lines in the solar spectrum. The main objection to the use of the above formula is the great amount of labor involved in making the solution for the constants, in order that the curve should represent accurately the region of spectrum employed.

Professor HARTMANN[2] proposed a dispersion formula which is quite simple and gives a curve which will represent with sufficient accuracy a long section of the spectrum. If x be the micrometer reading corresponding to the wave-length λ, then $x - x_0 = \dfrac{c}{(\lambda - \lambda_0)^a}$ where x_0, λ_0, c, and a are constants to be determined by observation. λ_0 and a characterize the general form of the dispersion curve and are for the same spectrograph nearly constant; c is the screw value, and is affected directly by a change in dispersion with temperature; x_0 is an additive constant which depends upon the position the plate is initially given on the measuring-engine. He determined the constant a in the following manner. Settings were made on five lines in different parts of a spectrogram of the Sun. Assuming $a = 1$, he computed from the two end lines and the middle line the values of the other constants. The residuals from the observed minus the computed micrometer readings on the other two lines were of opposite sign. An assumption of $a = 0.9$ gave smaller residuals for these two lines. From the residuals obtained, using these two values of a, it was evident that the correct value of a was about 0.6. Using this value of a, he then computed the micrometer settings for the other lines which he wished to use and found that this dispersion curve represented the observed settings within the errors of observation for the region λ 4220 to λ 4680. Beyond these points it runs off rapidly. HARTMANN computes, from plates made at different temperatures, tables for every 4° difference in temperature of the spectrograph, and uses the table nearest to the dispersion of the star plate for the reduction. With this method of procedure the correction for scale is practically negligible. The value of r is obtained by differentiation of the original formula and its value is given by $\dfrac{(\lambda - \lambda_0)^{1+a}}{a\,c}$ where the same constants are taken as in the original computation.

---

[1] *l. c.*, p. 142.

[2] This is generally known as the Cornu-Hartmann dispersion formula. The form in which it was originally given by CORNU was with the exponent $a = 1$. The formula was arrived at, however, independently by HARTMANN, by plotting the

Most line-of-sight observers have adopted the Cornu-Hartmann dispersion formula for the reduction of their spectrograms, although the details of the reductions vary. In the computation of the dispersion table for the new Mills spectro-

---

refractive index of quartz for different wave-lengths. This curve is an equilateral hyperbola which could be represented by an equation of the above form. See Potsdam *Publications*, Vol. XII.

graph we used the formula with $a = 1$, divided the spectrum from λ 4340 to λ 4630 into three sections, and computed a set of constants from three lines for each section. The continuity of the curve was assured by taking the last lines of the first and second sections as the first lines respectively of the second and third sections. The representation of the observations was found to be satisfactory. The dispersion table of the Southern Mills spectrograph was computed, placing $a = 1$, from only three lines for the whole region. The residual differences from the observed values, when plotted against the wave-lengths as abscissæ, gave an S-shaped curve from which corrections to the computed values were read off.

Professor FROST[1] prefers to reduce each of his plates independently of any other plate; *i. e.* he computes a dispersion table for each plate. Three lines of the comparison spectrum chosen for their sharpness and for their proper spacing (one near each end and one near the middle of the spectrum) give the values of the constants λ_0, x_0, and c in the Cornu-Hartmann formula where $a = 1$. He then computes the wave-lengths of the comparison and star lines measured. The differences between the computed values of the wave-lengths of the comparison lines and those given for the same lines in ROWLAND'S tables of solar wave-lengths are taken as "corrections to comparison lines." These, aside from the accidental errors of setting, he takes as representing the departure of the formula from an exact representation of the wave-lengths. The corrections for the star lines are interpolated from the corrections for the adjacent comparison lines. He prefers to do this rather than to draw a smooth curve for the corrections to the comparison lines, fearing the arbitrary smoothing out of such a curve more than the accidental errors of settings and the more serious effects of errors in wave-length of the comparison lines.

Methods of reduction which depend upon dispersion formulæ require an accurate knowledge of the wave-lengths of the lines used in both the comparison and stellar spectrum. Accurate values of the absolute wave-lengths are not required, but their relative values must be well determined. For example, a relative error of ± 0.01 tenth-meter in the wave-length of any line would produce an error in the velocity for that line of nearly a kilometer. The only system of wave-lengths which is at present available for our purpose is that due to ROWLAND, and this system has been generally adopted by line-of-sight observers. It has been shown,[1] however, that errors in relative wave-length exist in ROWLAND'S tables, amounting in some cases to as much as .01 or .02 tenth-meters. Furthermore the adjustment of the wave-lengths of the solar spectrum to those of the laboratory metallic spectra was not accomplished in a manner free from objections, so that systematic differences between the two result. Another difficulty arises, in the case of obtaining wave-lengths for stellar lines, due to the fact that stellar spectrographs have not sufficient resolution to separate lines which were measured as separate lines by means of the more powerful instrument used by ROWLAND. It is the practice of many observers, where two lines merge together to form one line in the star spectrum, to take the mean of the wave-lengths of the component lines weighted according to the intensities given by ROWLAND for those lines in the Sun. Wave-lengths based on estimates of intensities should naturally be regarded with suspicion, and in fact we do not know until the entire plate has been reduced whether we have chosen an erroneous wave-length or not. After we have reduced a number of plates of stars of the same type, we can correct the wave-lengths of those lines which consistently give residuals of the same sign. Comparison lines which consistently fall off the curve drawn for the other comparison lines are corrected in like manner.

---

[1] *Publications* of the Yerkes Observatory, Vol. II, 1903.

It is known, too, that various stellar lines and blends behave differently for stars of different types.[2] It is assumed that lines in solar-type stars have the same wave-lengths as similar lines in the Sun. In the case of other types the solar lines which occur can be used in determining the wave-lengths of the non-solar lines and blends. In this way special tables are constructed for stars of different types.

The two methods of measurement and reduction which follow eliminate the sources of errors incident to the above methods as far as it is possible to do so. The first is that due to Mr. R. H. CURTISS,[1] and is called by him "The Velocity Standard Method."

---

[1] See articles by FABRY and PEROT, *Astrophysical Journal*, Vol. 15, 270, 1902; EBERHART, Ibid., Vol. 17, 141, 1903; and HARTMANN, Ibid., Vol. 18, 167, 1903.

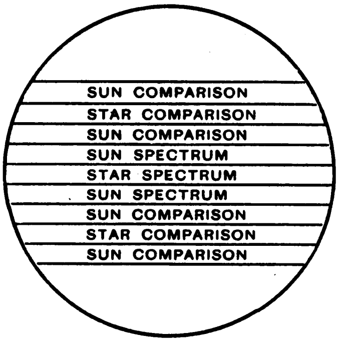
[2] S. ALBRECHT, *Astrophysical Journal*, Vol. 24, 333, 1906.

II. *The Velocity Standard Method.*—The method is in brief as follows: A standard velocity plate (*i. e.* a plate of a source whose velocity is accurately known), and the stellar plates are produced, as nearly as possible, under the same conditions of instrument, exposure, comparison spectra, etc. The measures of this standard plate fix the relative positions of the lines of the spectrum of the source and those of the comparison spectrum for a known velocity. These settings we call our zero velocity table. Now, measure the same lines, comparison and stellar, on the plate of the star whose velocity is to be determined. By a plot of the differences between the settings on the comparison lines of the standard and stellar plates the star plate is reduced to the dispersion of the standard plate. If the corrections from this plot are applied to the star lines, the differences between the measured positions of corresponding Fraunhofer lines in the standard and star spectra will be proportional to the difference of radial velocity of the sources producing them. It is usual to take a plate of the Sun or sky spectrum as the standard velocity plate, or several plates of either, all reduced to the same dispersion, from which to construct a zero-velocity table. It will be seen that the accurate wave-lengths of the lines of either comparison or stellar spectrum are not required in the above process of reductions, and in fact it is necessary to know them only roughly (to about 0.1 tenth-meter) for the computation of the factor rV_s. The assumption of the method is fundamental to all methods,—namely, that for solar-type stars the wave-lengths of the lines are the same as those of the corresponding lines in the Sun. For stars of other types it is necessary to form special tables for each type. The use of blends is based upon the assumption that the character of a blended line in the spectrum of a solar type star is the same as in the Sun, and is not dependent upon the estimated intensities of the component lines made by other observers. This freedom in the use of blends renders the method of great value not only to those using high-dispersion spectrographs but especially to those using low-dispersion instruments.

---

[1] CURTISS, *Astrophysical Journal*, Vol. 20, 149, 1904; *L. O. Bulletin* No. 62, 1904.

III. *The Spectrocomparator.*[1]—The last method to be discussed and the one most recent, is that due to Professor Hartmann. It is in principle the same as the preceding method, except that the star plate is referred to the standard plate directly on the engine, in place of using a table of settings obtained from the standard plate. In order to accomplish this, he has designed a special form of measuring-engine known as the spectrocomparator. The instrument is provided with two plate carriages, one of which is movable. On one of the carriages the star plate is placed, and on the other, which is provided with a fine micrometer screw, is placed a standard plate of the Sun (obtained with the stellar spectrograph). The microscope has two objectives so arranged that the images of portions of the two plates are brought, by means of total reflecting prisms and a reflecting surface, to focus in the same plane and in the field of one eye-piece. Three strips of the surface of one of the prisms are silvered. These act as diaphragms in the path of rays coming from the Sun plate and as reflectors in the path of rays coming from the star plate. One of them cuts out the central strip of the Sun spectrum and throws into its place the central strip of the star spectrum. The other two cut out central strips of the comparison spectra of the Sun plate and throw into their places central strips from the comparison spectra of the star plate. The arrangement of the spectra in the field of view is as shown in the diagram.



By changing the relative magnifying powers of the two objectives he is able to produce the effect of making the dispersion of the two plates the same. The method of measurement is, then, after proper alignment of the plates, to bring corresponding sections of the two plates into the field of the microscope, set the corresponding lines of the two comparison spectra in the same straight line, and read the micrometer head. The Sun plate is then moved along by the micrometer-screw until corresponding lines in the solar spectrum are in the same straight line with those of the star spectrum. The difference of the readings in the two positions is the displacement of the star lines relative to the solar lines, and is proportional to the difference in radial velocities of the star and Sun. The spectrum is divided into several sections for each of which these settings are made. The plates are then reversed and the measures repeated, care being taken that the second measure is made at a point of the screw 180° from that of the first. This method of procedure eliminates the physiological error dependent upon the positions of the plates and the periodic errors of the screw. The mean of the displacements in the two positions multiplied by the rV_s (he calls it s) for each section gives for each the value $V_* - V_0$, where V_* is the radial velocity of the star and V_0 the radial velocity of the Sun. In taking the mean,[1] the values $V_* - V_0$ for the different sections are not of equal weight, since the displacements in the violet region of the spectrum, where the dispersion is greater, are measured with a higher percentage of accuracy than those in the region of longer wave-lengths. Assume that the probable error of the measure of the line displacement is the same for all sections. Then the values $V_* - V_0 = sd$ should receive weights proportional to $\frac{1}{s}$ in taking the mean.

---

[1] Potsdam *Publications,* Vol. 18, 1906; also, *Astrophysical Journal,* Vol. 24, 285, 1906.

$$\text{The mean} = M_2 = \frac{\Sigma \frac{1}{s}\,(V_* - V_0)}{\Sigma \frac{1}{s}} = \frac{\Sigma\, d}{\Sigma \frac{1}{s}}$$

He now puts $f = \frac{1}{2\Sigma\frac{1}{s}}$ and since $\Sigma d = \frac{1}{2}\,(\Sigma d_1 + \Sigma d_2)$, where d_1 and d_2 are the displacements in the direct and reverse measure, it follows that $M_2 = f\,(\Sigma d_1 + \Sigma d_2)$. This leads to a very simple method of computation. Take the sum of the displacements in the direct and reverse measures and multiply by a factor which is a constant so long as the same regions are used and whose values are computed for all combinations of regions that are used. The product $= M_2$. This corrected for the velocity of the original Sun plate (V_0) gives the radial velocity of the star relative to the observer. The reduction to the Sun is made in the usual way.

---

[1] HARTMANN uses a camera lens giving good focus over a long range of spectrum. In stars of later types he is able to measure a length of spectrum from λ 4650 to λ 4060. In this extent of spectrum the change of dispersion is sufficient to make the above correction appreciable. It is negligible, however, when we are limited to a spectrum of only two or three hundred tenth-meters in length.

The great advantage claimed for the method, aside from those which it possesses in common with the velocity standard method, is that we are able to measure and reduce, in an hour or so, a plate of a star of a type rich in lines (several hundred on a plate) and practically utilize all the material on the plate. With the older methods to make such a measure and reduction utilizing all of the lines on the plate would require one or two days.

In Conclusion.—For the measure and reduction of spectrograms of stars of the earlier types, the use of the Cornu-Hartmann dispersion formula will suffice, in as much as the spectra of such stars consist of lines due to the simple gases, the wave-lengths of which have been accurately determined in the laboratory.

The measure and reduction of spectrograms of stars of the solar and later types will be accomplished with great saving of time and labor, and, moreover, by a method free from some of the uncertainties of wave-lengths by the use of the spectro-comparator. If the observer is not provided with such an instrument, the standard-velocity method would be used, in preference to any dispersion formula method, at least until a system of wave-lengths of the requisite accuracy is available.

For the case of later type stars it will be necessary to make corrections for the variation of lines with spectral type. It should be noted, however, that the corrections for some lines are positive while for others they are negative, so that the effects due to the variation of lines in stellar spectra of different types are to some extent compensating in radial velocity determinations.

MT. HAMILTON, CAL.